About Control Theory

Dr Mohammed Tikrity

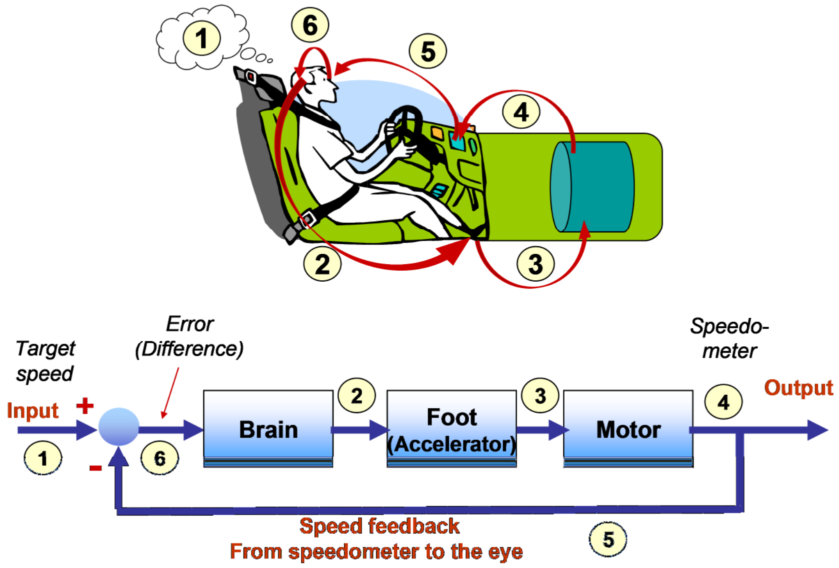
Automatic control is an enormously successful field that affects every aspect of our lives. A combination of technological developments, economic pressures, and research advances has promoted control into a central position in technology, and over the next several decades, the impact of automatic control systems will continue to grow. The applications we have seen so far–such as cheap and fast computer disk drives, active vehicle suspension control, fly-by-wire aircraft, highly integrated manufacturing facilities, and manned and unmanned space systems–are only the beginning of this trend.

Control theory concerns itself with means by which to alter the future behaviour of systems (clearly the past cannot be influenced nor, since no response can take place in any system in zero time, can the present). Furthermore, the objective of any control system in every case is connected with the performance of the system over some period of time, even though this may lead to conflicting requirements in the short term. For control theory to be successfully applied there needs to at least two possible actions at any stage in the control system as the system would follow an unchangeable course otherwise. In addition, control theory also needs access to some means of choosing the correct (or most applicable) actions that will result in the desired behaviour being produced.

The Concept of a Control Loop

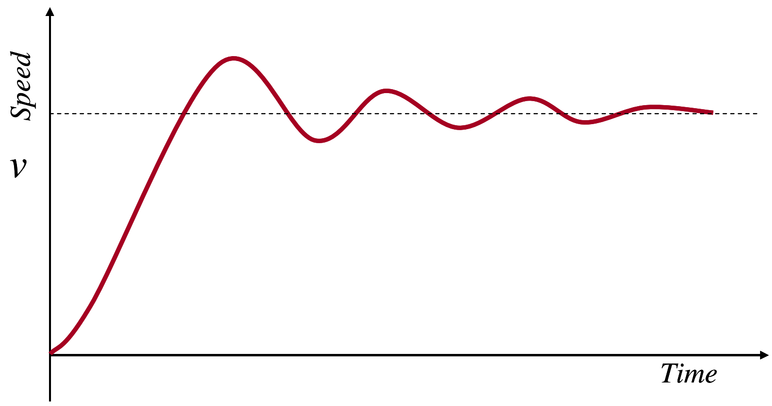
Control theory was developed to support the emergent activity of automatic control. A central idea of control theory is the *control loop*.

In the broadest form, a control loop operates on the principles of negative feedback. The effects of an action are reported to the controller through an information channel. The controller strives to minimize the error difference between the measured and the desired behaviours and commands the next action appropriately. A straight-forward example of this control concept is the cruise control feature of a car: if the measured speed of the car drops below the set speed (because of an uphill stretch), the cruise controller will accelerate; if the car rolls too fast, the controller will shift down in an attempt to minimize the discrepancy between the desired and the measured speeds.



The main performance measure for a control system relates to rate of error reduction. Usually, performance is quoted in terms of the highest frequency that the control system can follow, when required to do so. All control loops tend to become unstable as higher and higher performance is sought.

Referring back to the cruise control example, it is desirable to have the controller act against a speed change in a timely manner, but a sudden acceleration may well lead to an overshooting of the desired velocity, which the controller then has to quickly correct, possibly undershooting this time. Consequently, the car's speed will oscillate around the desired goal, which is a problem of loop-driven control theory known as *instability*.



The oscillating effects of instability on cruise control.

v represents the desired speed

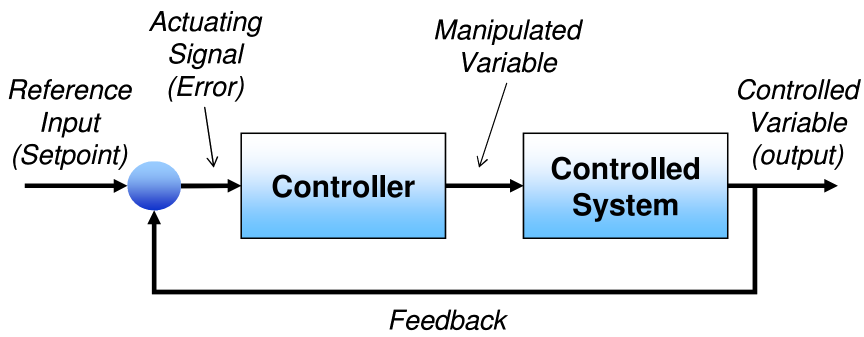
All control loops have the same basic form, regardless of the particular application area. Thus, control theory can be considered to concentrate on studying the universal situations that underlie all applications of quantitative control.

Feedback Control

The most fundamental element of any automatic control system is the basic feedback control loop. The concept of feedback control is not new; the first such industrial loop was applied in 1774 by James Watt for controlling the speed of an early steam engine. Today the application of feedback control loops is an essential element in successfully and economically manufacturing virtually every industrial product from home washing machines to space vehicles.

Feedback control is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. Feedback control may be defined as the use of difference signals, determined by comparing the actual values of system variables to their desired values, as a means of controlling a system. An everyday example of a feedback control system is an automobile speed control, which uses the difference between the actual and the desired speed to vary the fuel flow rate. Since the system output is used to regulate its input, such a device is said to be a *closed-loop control system*.

The following figure shows basic elements of a feedback control system as represented by a block diagram. The functional relationships between these elements are easily seen. An important factor to remember is that the block diagram represents flow-paths of control signals, but does not represent flow of energy through the system or process.



Basic elements of a feedback control system

The controller is a component needed to generate the appropriate control signal applied to the controlled system.

The reference point is an external signal applied to the summing point of the control system to cause the controlled system to produce a specified action. This signal represents the desired value of a controlled variable and is also called the "set-point."

The controlled output is the quantity or condition of the system which is controlled. This signal represents the controlled variable.

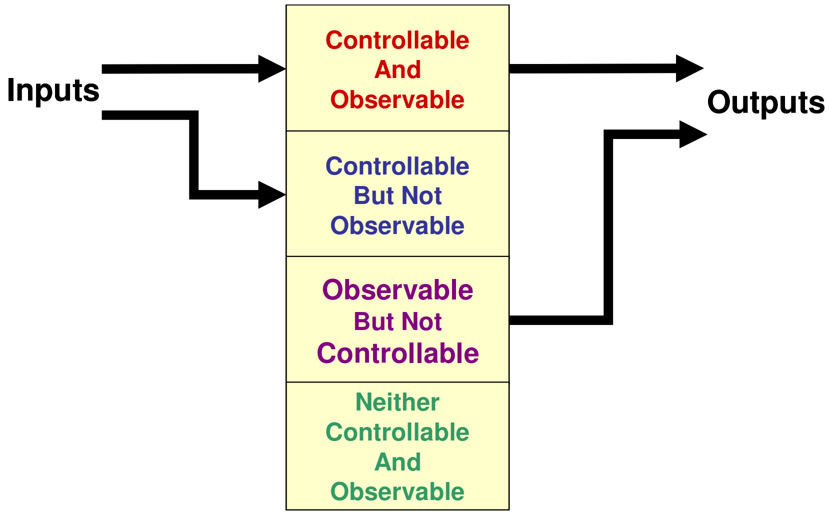
The feedback signal is a function of the output signal. It is sent to the summing point and algebraically added to the reference input signal to obtain the actuating signal.

The actuating signal represents the control action of the control loop and is equal to the algebraic sum of the reference input signal and feedback signal. This is also called the "error signal."

The manipulated variable is the variable of the process acted upon to maintain the system output (controlled variable) at the desired value.

Controllability and Observability

Controllability and Observability are fundamental properties of dynamic systems.



Controllability

The measure of controllability refers to the capability of the controlled object to move to a desired state. An object has a low degree of controllability if it cannot move to a desired state in one control step.

In a basic control system, controllability is a necessary precondition for control by feedback loops. When the controller detects a deviation of the measured behaviour from the expected behaviour, it needs to take immediate counter-action. Low degrees of controllability almost always translate to instability of the control system. Conceptually, one may compare this to a large ship, which has a great amount of inertia. If the ship's motors attempt to stop the ship right as it reaches its goal position, the ship will run over and its position will have to be readjusted because the ship's inertia causes it to have a very low degree of controllability. The solution would be to reverse the motors a mile or two before the ship reaches the goal position.

Observability

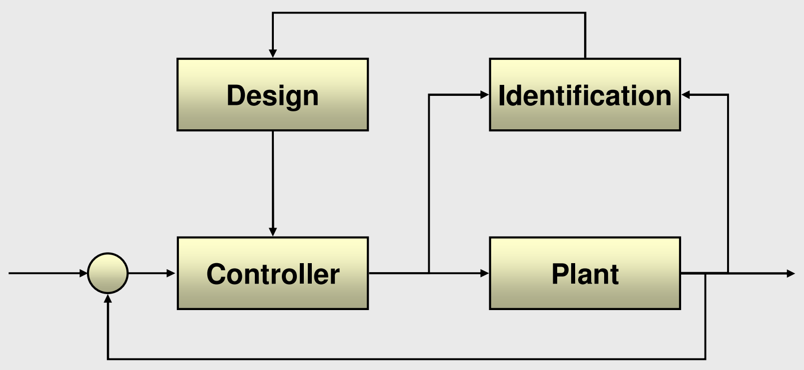
Opposite to controllability is observability, which refers to the accessibility of the controlled object's state in the environment. Assuming that there exist no limitations on the communication from the controlled object to the controller, observability mainly manifests in the object's capability to translate its environmental perception into a state vector. This translation process may involve the integration of different types of sensors, and therefore information, as well as dealing overlaps between sensor data. The controlled object, in the first place, needs to be able to extract the information out of its environment which the controller needs in order to instantiate the necessary control process.

Adaptive Control

“To adapt" means to change behaviour to conform to new circumstances. Intuitively, an adaptive controller is thus a controller that can modify its behaviour in response to changes in the dynamics of the process and the character of the disturbances.

Since the early 1970s, academics and industrial researchers have been working on feedback controllers that can learn about and adapt to gradual changes in the behaviour of the controlled process.

An adaptive controller can update its tuning parameters all the while it is in operation so that its performance remains optimal, even if the behaviour of the process changes. This amounts to automatically updating the controller’s entire strategy to accommodate the new behaviour of the process.



Block diagram of an adaptive control system

In the figure above the upper loop constituted by the blocks denoted "identification" and "design" is what separates the adaptive controller from a conventional one. The identification block contains some kind of recursive estimation algorithm which aims at determining the best model of the process at the current instant. The interplay between identification and control is the core issue in any adaptive control method. The design block then applies this model to produce a controller according to some strategy or criteria.

Reinforcement Learning and Adaptive Control

Adaptive control is also concerned with algorithms for improving a sequence of decisions from experience. Adaptive control is a much more mature discipline that concerns itself with dynamic systems in which states and actions are vectors and system dynamics are smooth: linear or locally linearizable around a desired trajectory.

Although the dynamic model of the system is not known in advance, and must be estimated from data, the *structure* of the dynamic model is fixed, leaving model estimation as a parameter estimation problem. These assumptions permit deep, elegant and powerful mathematical analysis, which in turn lead to robust, practical, and widely deployed adaptive control algorithms.

There are two types of adaptive control - *Indirect Adaptive Control* in which the controller is based on a model of the plant and estimation of parameters in the model leads indirectly to adaptation of parameters in the controller; *Direct Adaptive Control* where no plant parameter estimation takes place but instead certain information about the plant is used to find suitable ways for convergent adaptation of the controller parameters directly.

Using Mathematical Models

Within the domain of engineering, there must exist an accurate mathematical model of the behaviour to be exhibited by the control process. Given a mathematically defined model, the controlling entity is capable of comparing different situations (as established in the forward model) with respect to the desired goal state so as to be able to choose the one that will get the controlled object closer to the goal state in terms of the accurate mathematical model.

System identification

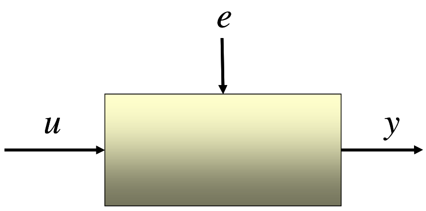
System identification may be defined as the process of determining a model of a dynamic system using observed system input-output data. The identification of dynamic systems through the use of experimental data is of considerable importance in engineering since it provides information about system parameters which is useful in predicting behaviour and evaluating performance.

System Identification allows us to build mathematical models of a dynamic system based on measured data. This can be done by adjusting parameters within a given model until its output coincides as well as possible with the measured output.

System identification can be accomplished by two approaches. One is based on the physical principles underlying the phenomenological behaviour of the process and the other is called *black-box modelling* in which a discrete time model is chosen and its parameters are estimated by fitting the input-output data. In the former approach, applying the physical laws governing the process, the basic relations are written as equations representing balance of certain physical entities in the process.

Physical system modelling gives rise to generic models which are native to the continuous-time domain and the numerical values of the parameters in such models can be directly estimated from input-output data using the techniques of identification that are specially developed for continuous-time models in the recent decades. In certain situations, the essential features of the behaviour of a system can be quickly obtained without many details by means of experiment.

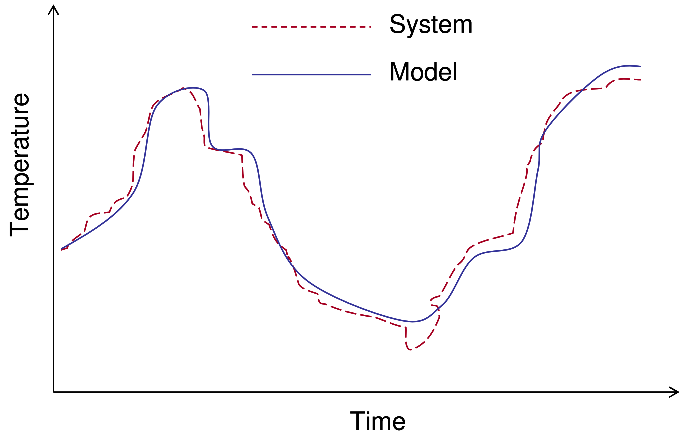
Models describe relationships between measured signals. It is convenient to distinguish between *input* signals and *output* signals. The outputs are then partly determined by the inputs. Think for example of an airplane where the inputs would be the different control surfaces, ailerons, elevators, and the like, while the outputs would be the airplane's orientation and position. In most cases, the outputs are also affected by more signals than the measured inputs. In the airplane example it would be wind gusts and turbulence effects. Such “unmeasured inputs" will be called *disturbance* signals or *noise*. If we denote inputs, outputs, and disturbances by *u*, *y*, and *e*, respectively, the relationship can be depicted in the following figure.



Input Signals u, Output Signals y, and Disturbances e

All these signals are functions of time, and the value of the input at time *t* will be denoted by *u(t)*. Often, in the identification context, only discrete-time points are considered, since the measurement equipment typically records the signals just at discrete-time instants, often equally spread in time with a sampling interval of *T* time units. The modelling problem is then to describe how the three signals relate to each other.

A good test if the model is any good is to take a close look at the model's output compared to the measured one. The following diagram shows the output of a real system (dotted lines) and the output of a model (continuous line) obtained by identification process.



To identify a black box system a random signal (called pseudo Random Signal or PRS) input is used. This input excites the system. From the measurements of the input and output signal the system parameters can be identified and the system model can be built.

Variants of Model Descriptions

Linear state-space models are also easy to work with. The essential structure variable is just a scalar: the model order. This gives just one knob to turn when searching for a suitable model description. See below.

General linear models can be described symbolically by

$$Y = Gu + He$$

which says that the measured output *y(t)* is a sum of one contribution that comes from the measured input *u(t)* and one contribution that comes from the noise *He*. The symbol *G* then denotes the dynamic properties of the system, that is, how the output is formed from the input. For linear systems it is called the *transfer function* from input to output. The symbol *H* refers to the noise properties, and is called the *disturbance model*. It describes how the disturbances at the output are formed from some standardized noise source *e(t)*.

State-space models are common representations of dynamical models. They describe the same type of linear difference relationship between the inputs and the outputs, but they are rearranged so that only one delay is used in the expressions. To achieve this, some extra variables, the *state variables*, are introduced. They are not measured, but can be reconstructed from the measured input-output data.

The procedure to determine a model of a dynamical system from observed input-output data involves three basic ingredients:

- The input-output data
- A set of candidate models (the model structure)

- A criterion to select a particular model in the set, based on the information in the data (the identification method)

The identification process amounts to repeatedly selecting a model structure, computing the best model in the structure, and evaluating this model's properties to see if they are satisfactory.

1. Transfer Function Model
2. Impulse Response Model
3. The State Space Model
4. The Input-Output Model

Description of Dynamic Systems

Control System Professional deals with state-space and transfer function models of continuous-time (analogue) and discrete-time (sampled) systems. Transfer Function Representations

- State-Space Representations
- Continuous-Time versus Discrete-Time Systems
- Discrete-Time Models of Continuous-Time Systems
- Discrete-Time Models of Systems with Delay

State-Space Representations

There are several different ways to describe a system of linear differential equations. The *state-space representation* is given by the equations:

$$\frac{\mathbf{d}\vec{\mathbf{x}}}{\mathbf{dt}} = \mathbf{A}\vec{\mathbf{x}} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{C}\vec{\mathbf{x}} + \mathbf{Du}$$

where $\mathbf{x}$ is an n by 1 vector representing the state (commonly position and velocity variables in mechanical systems), $\mathbf{u}$ is a scalar representing the input (commonly a force or torque in mechanical systems), and $\mathbf{y}$ is a scalar representing the output. The matrices $\mathbf{A}$ (n by n), $\mathbf{B}$ (n by 1), and $\mathbf{C}$ (1 by n) determine the relationships between the state and input and output variables. Note that there are n first-order differential equations. State space representation can also be used for systems with multiple inputs and outputs (MIMO).

Standard Models

Standard models are very useful for structuring our knowledge. It also simplifies problem solving. There are four standard forms

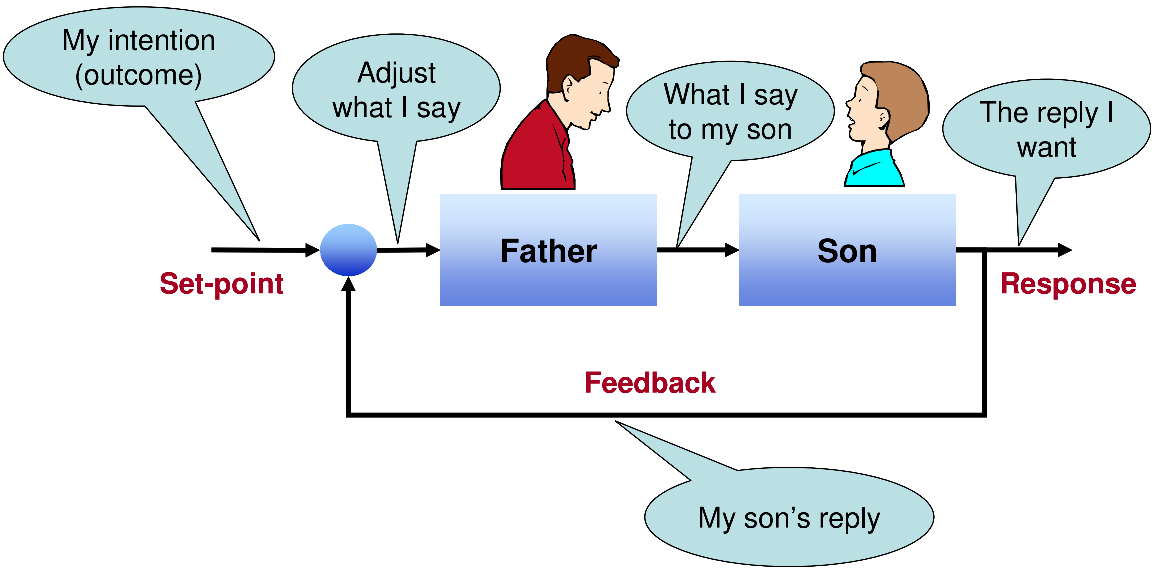
- Ordinary differential equations
- Transfer functions
- Frequency responses
- State equations

The first two standard forms are primarily used for linear time-invariant systems. The state equations also apply to nonlinear systems.

NLP and Control Theory

Can NLP benefit from Control Theory (CT)? The answer is certainly yes. There are many analogies between NLP and CT. Although human being is a very complex system but he is still a system.

Consider a subject as a system with input, output and noise as follows:



Communication is a closed loop system

Communication between an NLP practitioner and a subject is a closed loop system. The practitioner acts as a controller, and the subject as a controlled system. Practitioner has a certain outcome (set-point in CT). He observes the external behaviour of the subject (feedback in CT), compares the behaviour with his outcome (signal addition in CT).

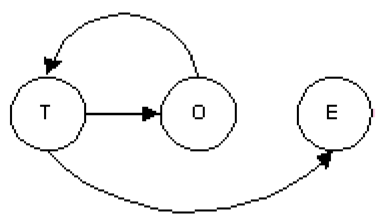
The following table compares some similar concepts in Control Theory and NLP:

Control Theory	NLP	Notes
Set point	Outcome	
State	State	
Initial State	Current State	
Desired State	Desired State	
Input	Stimulus	
Output	Response	
Identification	Elicitation	
Modelling	Modelling	
Feedback	Feedback	
Control Algorithm	Strategy	
Control Design	Strategy Design	
Control variable	Installation	

TOTE Model

TOTE model developed by Miller, Galanter and Pibram in 1960. It stands for the sequence Test – Operate –Test - Exit, which describes the basic feedback loop used to guide all behaviour.

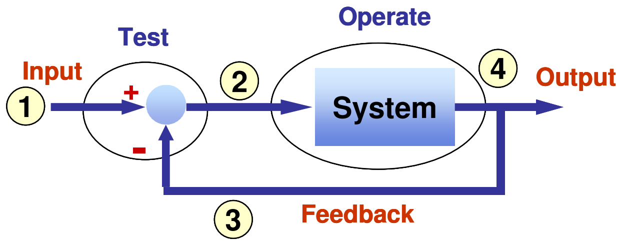
Miller et al. suggested that TOTE should replace the stimulus-response as the basic unit of behaviour. In a TOTE unit, a goal is tested to see if it has been achieved and if not an operation is performed to achieve the goal; this cycle of test-operate is repeated until the goal is eventually achieved or abandoned.



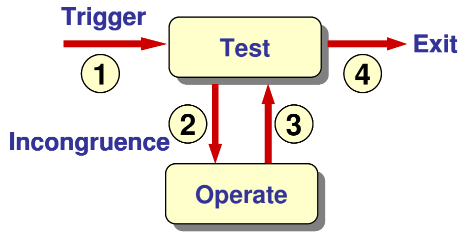
The TOTE Model as a Graph

The stimulus behind behaviour is the achievement of a Goal. In order to achieve the Goal, that goal has to be defined thoroughly enough to allow us to recognise when the goal has been achieved so that as we move towards the achievement of that goal (operate) we can assess (test) if that goal has been achieved and then Exit.

George Miller intended this to be a model of behaviour, not a model of IT. Is it possible that I am trying to forcefully overlay one model on top of another just because it has the word "test" in it, not once but twice?



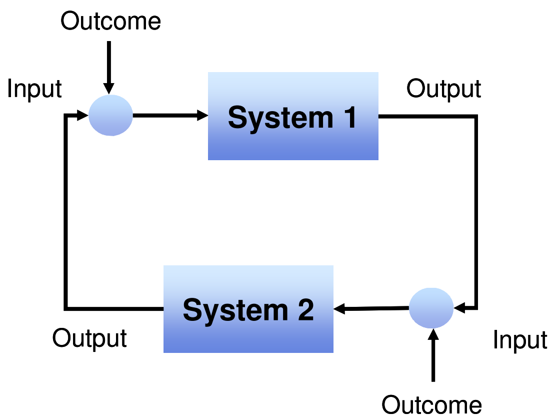
CONTROL SYSTEM Representation



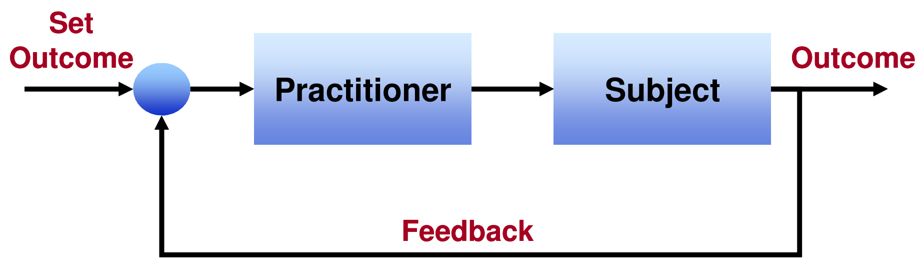
TOTE Representation

Examples

Discussion

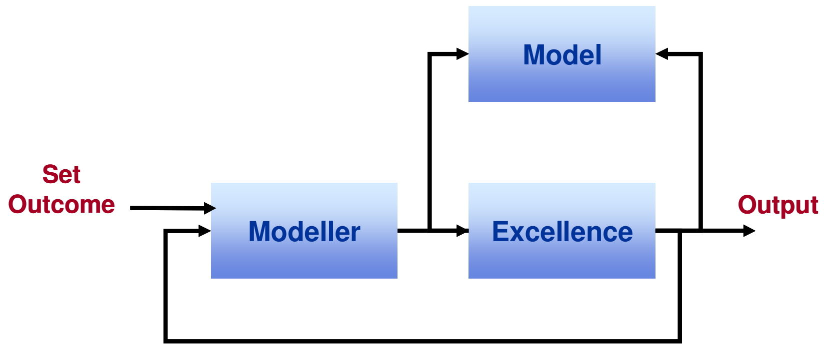


Therapy



Practitioner as a "controller"
Connection represents Rapport

Modelling



Practitioner as a "Modeller"